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Gluon mini-jet production in nuclear collisions at high energies

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Abstract

We show explicitly that the leading soft gluon p_T distribution, predicted by Kovner, McLerran, and Weigert after solving classical Yang-Mills equations, can be understood in terms of conventional QCD perturbation theory. We also demonstrate that the key logarithm in their result represents the logarithm in DGLAP evolution equations.

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I. INTRODUCTION

In ultra-relativistic heavy ion collisions, physical observables sensitive to a few GeV momentum scale, such as the mini-jet production, will be dominated by scattering of soft gluons from both heavy ion beams. Understanding the distribution of soft gluons formed in the initial stage of the collision is particularly interesting and important. In terms of conventional QCD perturbation theory, a calculable cross section in high energy hadronic collisions is factorized into a single collision between two partons multiplied by a probability to find these two partons of momentum fractions x_1 and x_2 , respectively, from two incoming hadrons. The probability is then factorized into a product of two parton distributions $\phi(x_1)$ and $\phi(x_2)$, which are probabilities to find these two partons from the respective hadrons [1]. Because of extremely large number of soft gluons in heavy ion beams, it is natural to go beyond the factorized single-scattering formalism to include any possible multiple scattering, and long range correlations between soft gluons from two incoming ions.

Recently, McLerran and Venugopalan (MV) developed a new formalism for calculation of the soft gluon distribution for very large nuclei [2,3]. In this approach, the valence quarks in the nuclei are treated as the classical source of the color charges. They argued that the valence quark recoil can be ignored in the limit when the gluons emitted are soft. In addition, because of the Lorentz contraction, the color charge of the valence quarks is treated approximately as an infinitely thin sheet of color charge along the light cone. With these assumptions, the gluon distribution function for very large nuclei may be obtained by solving the classical Yang-Mills Equation [3,4]. Using the classical glue field generated by a single nucleus obtained in the MV formalism as the basic input, Kovner, McLerran, and Weigert (KMW) computed the soft gluon production in a collision of two ultra-relativistic heavy nuclei by solving the classical Yang-Mills equations with the iteration to the second order [5]. The two nuclei are treated as the infinitely thin sheets of the classical color charges moving at the speed of light in the positive and the negative z directions, respectively. Following this approach, the distribution of soft gluons at the rapidity y and the transverse momentum p_T in nuclear collisions can be express as [5]

$$\frac{dN}{dyd^2p_T} = S_T \frac{2g^6\mu^4}{(2\pi)^4} N_c(N_c^2 - 1) \frac{1}{p_T^4} \ln \left(\frac{p_T^2}{\Lambda_{cutoff}^2} \right), \quad (1)$$

where g is the strong coupling constant, $N_c = 3$ is the number of the color, and Λ_{cutoff} is a cutoff mass scale [5]. Note that there was a factor of π misprint in Eq. (50) of Ref. [5], as pointed out in Ref. [6]. In Eq. (1), μ^2 is the averaged color charge squared per unit area of the valence quark, and S_T is the transverse area of the nuclei. The μ^2 and the S_T are related as [6]

$$S_T \mu^2 = \frac{N_q}{2N_c}, \quad (2)$$

where N_q is the number of valence quarks in the color charge source. The number distribution in Eq. (1) can be also expressed in terms of the cross section [6]

$$\frac{d\sigma}{dyd^2p_T} = \frac{2g^6}{(2\pi)^4} \left(\frac{N_q}{2N_c} \right)^2 N_c(N_c^2 - 1) \frac{1}{p_T^4} \ln \left(\frac{p_T^2}{\Lambda_{cutoff}^2} \right). \quad (3)$$

In deriving Eq. (3), the following relation was used [6]

$$\frac{d\sigma}{dyd^2p_T} = S_T \frac{dN}{dyd^2p_T} . \quad (4)$$

The key result derived in Ref. [5], Eq. (1) (or equivalently, Eq. (3)), is potentially very useful in estimating the production of mini-jet rates, and the formation of the possible quark-gluon plasma at RHIC [7]. The purpose of this paper is to understand the respective role of perturbative and non-perturbative QCD in deriving the expression in Eq. (1) (or that in Eq. (3)), and explore under what kind of approximation this result matches the conventional perturbative calculation.

KMW's derivation for Eq. (1) is based on the following physical picture: in ultra-relativistic heavy ion collisions, gluons are produced by the fields of two strongly Lorentz contracted color charge sources, which are effectively equal to the valence quarks of two incoming ions. In order to understand KMW's result in terms of the language of perturbative QCD, we consider a specific partonic subprocess: $qq \rightarrow qqq$, as sketched in Fig. 1. If we assume that the incoming quarks qq are the valence quarks in the initial color charge sources, the partonic subprocess in Fig. 1 mimics the physical picture adopted in KMW's derivation. However, as a Feynman diagram in QCD perturbation theory, the single diagram shown in Fig. 1 is not gauge invariant. As we demonstrate in Sec. II, under certain approximations, the contributions *extracted* from the diagram in Fig. 1 to the leading soft gluon production in Eq. (1) is gauge invariant; and therefore, the physical picture proposed by KMW for soft gluon production is preserved.

In Sec. III, within the framework of conventional perturbative QCD, we calculate the gluon production through the partonic subprocess $qq \rightarrow qqq$, as shown in Fig. 1, at the soft gluon limit. With our explicit calculation of this subprocess, we demonstrate that Eq. (1) (or the cross section in Eq. (3)) at $N_q = 1$ can be reproduced. Through our derivation, we show that the key logarithm $\ln(p_T^2/\Lambda_{cutoff}^2)$ in Eq. (1) (or in Eq. (3)) is basically the logarithm from the splitting of the incoming quark to the soft gluon in Fig. 1. In terms of the conventional QCD factorization formalism [1], such logarithm is normally factorized into the distributions of the collinear gluons inside the incoming hadrons, and the logarithmic dependence of the distributions is a direct result of solving the DGLAP evolution equations [8].

Finally, in Sec. IV, we discuss the relations between the MV formalism and that of the conventional QCD factorization. We explicitly demonstrate that KMW's result can be reduced to the factorized formula in the conventional perturbative QCD, if we replace the charge density for the classical color charge μ^2 (or equivalently N_q) by the valence quark distributions of the nuclei, and absorb the logarithm $\ln(p_T^2/\Lambda_{cutoff}^2)$ into one of the valence quark distributions. We point out that with the higher order of iteration, KMW's approach may include the multi-parton dynamics which is not apparent in the conventional perturbative calculation.

II. FACTORIZATION AND GAUGE INVARIANCE

As we discussed above, the partonic process $qq \rightarrow qqq$, as shown in Fig. 1, mimics the physical picture adopted in KMW's derivation, if we assume that the incoming quarks qq

are the valence quarks in the initial color charge sources. However, in general, the Feynman diagram shown in Fig. 1 is not gauge invariant by itself. In this section, we discuss how to extract the gauge invariant leading contribution from the diagram in Fig. 1, and what kind of approximation we need to take in order to extract such leading contributions.

For the production of gluons, we evaluate the invariant cross section, $d\sigma_{qq \rightarrow g}/dyd^2p_T$, with y and p_T the rapidity and the transverse momentum of the produced gluon. We label l_1 and l_2 as the momenta of the two incoming quarks, respectively, and we choose k_1 and k_2 to be the momenta for the two gluons emitted from the initial quarks. We have $p^2 = 0$ for the final-state gluon because of its on-shell condition. For the other two gluons, k_1 and k_2 can not be on shell at the same time, because k_1 and k_2 come from different directions, and $p^2 = (k_1 + k_2)^2 = 0$.

In general, the cross section can be written as

$$d\sigma = \frac{1}{2s} |\overline{M}|^2 dps , \quad (5)$$

where $s = (l_1 + l_2)^2$, and $|\overline{M}|^2$ is matrix element square with the initial-spin averaged and the final-spin summed. In Eq. (5), dps is the phase space, and can be expressed as:

$$\begin{aligned} dps &= \frac{d^4 k_1}{(2\pi)^4} (2\pi) \delta((l_1 - k_1)^2 - m^2) \times \frac{d^4 k_2}{(2\pi)^4} (2\pi) \delta((l_2 - k_2)^2 - m^2) \\ &\quad \times \frac{d^3 p}{(2\pi)^3 2E} (2\pi)^4 \delta^4(k_1 + k_2 - p) \\ &= \frac{d^3 p}{E} \frac{1}{2(2\pi)^3} \frac{d^4 k_1}{(2\pi)^4} (2\pi)^2 \delta((l_1 - k_1)^2 - m^2) \delta((l_2 - k_2)^2 - m^2) , \end{aligned} \quad (6)$$

where $k_2 = p - k_1$, and m is the quark mass. For simplicity, we assume that both incoming quarks have the same mass. In high energy collisions, we set the mass of light quarks to be zero. Because of the gluon propagators, as shown in Fig. 1, the matrix element square $|\overline{M}|^2$ has the following pole structure:

$$poles = \frac{1}{k_1^2 + i\epsilon} \frac{1}{k_1^2 - i\epsilon} \frac{1}{k_2^2 + i\epsilon} \frac{1}{k_2^2 - i\epsilon} . \quad (7)$$

When integrating over the phase space, we see that the leading contribution comes from the terms with $k_1^2 \rightarrow 0$ or $(p - k_1)^2 = k_2^2 \rightarrow 0$ limit. As pointed out above, k_1^2 and k_2^2 can not be zero at the same time. Therefore, to calculate the leading contribution, we can first calculate the diagram in $k_1^2 \rightarrow 0$ limit. The total leading contribution is just twice of it, because the diagram is symmetric for k_1 and k_2 .

When we take $k_1^2 \rightarrow 0$, the integration become divergent. Therefore, an introduction of a cutoff is necessary for obtaining a finite contribution from the diagram in Fig. 1, and the corresponding contributions are sensitive to the cutoff. To derive the leading contribution at $k_1^2 \rightarrow 0$ limit, we perform the collinear approximation $k_1 \approx xl_1 + O(k_{1T})$, with $k_{1T} \sim \Lambda_{cutoff} \ll p_T$, where Λ_{cutoff} is a collinear cutoff scale [9]. This approximation means that the leading contribution is from the phase space where almost all transverse momentum of the final-state gluon comes from the gluon of k_2 , and k_1 is almost collinear to l_1 . After such collinear approximation, the cross section in Eq. (5) can be approximately written in a factorized form [1]:

$$E \frac{d\sigma_{qq \rightarrow g}}{d^3 p} \approx 2 \left(\frac{1}{2(2\pi)^3} \frac{1}{2s} \right) \int \frac{dx}{x} P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T) H(xl_1, l_2, p) + O\left(\frac{\Lambda_{cutoff}^2}{p_T^2}\right), \quad (8)$$

where the overall factor of 2 is due to the fact that the leading contribution come from two regions corresponding to $k_1^2 \rightarrow 0$ and $k_2^2 \rightarrow 0$, respectively. In Eq. (8), $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ represents the probability of finding an almost collinear gluon with the momentum fraction x from an incoming quark of momentum l_1 , and

$$P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T) = \int \frac{d^4 k_1}{(2\pi)^4} x \delta(x - \frac{k_1}{l_1}) |\bar{M}_{q \rightarrow g}|^2 (2\pi) \delta((l_1 - k_1)^2 - m^2). \quad (9)$$

The diagram for $|\bar{M}_{q \rightarrow g}|^2$ can be represented by Fig. 2. $H(xl_1, l_2, p)$ in Eq. (8) is effectively the hard scattering between the gluon of $k_1 = xl_1$ and the incoming quark of l_2 . and given by

$$H(xl_1, l_2, p) = \hat{H}(xl_1, l_2, p) (2\pi) \delta((l_2 + xl_1 - p)^2 - m^2), \quad (10)$$

where $\hat{H}(xl_1, l_2, p)$ is given by the diagrams shown in Fig. 3.

In addition to the diagram in Fig. 1, in general, we also need to consider the radiation diagrams shown in Fig. 4. Similarly, the contribution of Fig. 4a and Fig. 4b can also be written in the same factorized form:

$$E \frac{d\sigma_{qq \rightarrow g}^{\text{rad}}}{d^3 p} \approx \frac{1}{2(2\pi)^3} \frac{1}{2s} \int \frac{dx}{x} P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T) H_i(xl_1, l_2, p) + O\left(\frac{\Lambda_{cutoff}^2}{p_T^2}\right), \quad (11)$$

with $i = a, b$. Here $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ is defined by Eq. (9). $H_a(xl_1, l_2, p)$ and $H_b(xl_1, l_2, p)$ are the hard scattering parts from the diagrams in Fig. 4a and Fig. 4b, and they are represented by Fig. 5a and Fig. 5b, respectively. With the contribution from Fig. 4a and Fig. 4b, Eq. (8) changes to

$$\begin{aligned} E \frac{d\sigma_{qq \rightarrow g}}{d^3 p} \approx & 2 \left(\frac{1}{2(2\pi)^3} \frac{1}{2s} \right) \int \frac{dx}{x} P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T) \\ & \times [H(xl_1, l_2, p) + H_a(xl_1, l_2, p) + H_b(xl_1, l_2, p) + \text{interference terms}] \\ & + O\left(\frac{\Lambda_{cutoff}^2}{p_T^2}\right), \end{aligned} \quad (12)$$

with the approximation $k_1 = xl_1 + O(k_{1T})$.

Feynman diagrams shown in Fig. 3 and Fig. 5 form a gauge invariant subset for calculating the hard scattering parts, $H(xl_1, l_2, p)$'s in Eq. (12). Fig. 5a and Fig. 5b are effectively the s -channel and u -channel diagrams for the $gq \rightarrow gq$ partonic process. Since we are only interested in the soft gluon limit, when $|t| \ll s$, the contribution from these two diagrams can be neglected, in comparison to the contribution from the diagram in Fig. 3. In addition, under the collinear expansion $k_1 \approx xl_1$, the gluon line which connects the partonic parts $P_{l_1 \rightarrow k_1}$ and $H(xl_1, l_2, p)$ is effectively on the mass-shell, and therefore, the partonic parts, P and H in Eq. (8) are separately gauge invariant.

Similar arguments can be held for the situation when $k_2^2 \sim 0$ or $k_{2T} \ll p_T$. For example, after the collinear expansion for k_2 , the contributions from diagrams shown in Fig. 4c and Fig. 4d can be neglected in the soft gluon limit.

Therefore, with the approximation of $k_1^2 \sim 0$ (or $k_2^2 \sim 0$) and the soft gluon limit, and a proper choice of the gauge, the dominant contribution for the partonic process $qq \rightarrow qg$ comes from the diagram shown in Fig. 1. In the next section, we derive the leading contribution of the partonic process $qq \rightarrow qg$ with the above approximation.

III. DERIVATIONS

Following the discussion in last section, we now turn to explicit calculation of the leading contribution to the gluon production from the partonic process $qq \rightarrow qg$, shown in Fig. 1. As shown in Eq. (8), the leading contribution of this partonic process can be factorized into two parts: $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ and $H(xl_1, l_2, p)$. $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ represents the splitting of the quark to the soft gluon with momentum fraction x , and $H(xl_1, l_2, p)$ represents the scattering between the gluon of momentum xl_1 and the other incoming quark of momentum l_2 . $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ and $H(xl_1, l_2, p)$ are represented by the diagrams in Fig. 2 and Fig. 3, respectively. In the following derivation, we choose the center of mass frame, with

$$l_1 = (l_{1+}, l_{1-}, l_T) = (l_+, 0, 0) \quad \text{and} \quad l_2 = (0, l_-, 0). \quad (13)$$

The definitions of the plus and minus components of the four momentum $p = (p_0, p_1, p_2, p_3)$ are:

$$p_+ = \frac{p_0 + p_3}{\sqrt{2}}, \quad p_- = \frac{p_0 - p_3}{\sqrt{2}}. \quad (14)$$

We also introduce two useful vectors, n and \bar{n} as:

$$n = (0, 1, 0_T), \quad \bar{n} = (1, 0, 0_T). \quad (15)$$

As we discussed above, $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ and $H(xl_1, l_2, p)$ are separately gauge invariant. To derive the complete leading contribution, we choose $n \cdot A = 0$ gauge to calculate $P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T)$ and $\bar{n} \cdot A = 0$ gauge for calculating the $H(xl_1, l_2, p)$. From Eq. (9) and the diagram shown in Fig. 2, we have in $n \cdot A = 0$ gauge,

$$\begin{aligned} P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T) &= C_{q \rightarrow g} g^2 \int \frac{d^4 k_1}{(2\pi)^4} x \delta(x - \frac{k_1}{l_1}) (2\pi) \delta((l_1 - k_1)^2 - m^2) \\ &\times \frac{1}{2} \text{Tr}(\gamma \cdot l_1 \gamma^\alpha \gamma \cdot (l_1 - k_1) \gamma^\beta) \frac{P_{\alpha\mu}(k_1)}{k_1^2} \frac{P_{\beta\nu}(k_1)}{k_1^2} (-g^{\mu\nu}), \end{aligned} \quad (16)$$

where $C_{q \rightarrow g}$ is the color factor. In Eq. (16), the gluon polarization tensor is defined as

$$P_{\alpha\mu}(k_1) = -g_{\alpha\mu} + \frac{k_{1\alpha} n_\mu + n_\alpha k_{1\mu}}{k_1 \cdot n}. \quad (17)$$

The four dimension integral $d^4 k_1 = dk_{1+} dk_{1-} \cdot \pi dk_{1T}^2$. We can use the δ -function $\delta(x - \frac{k_1}{l_1})$ to fix k_{1+} , and use $\delta((l_1 - k_1)^2 - m^2)$ to fix k_{1-} . We have

$$k_{1+} = xl_+, \quad k_{1-} = -\frac{k_{1T}^2}{2l_+(1-x)}. \quad (18)$$

Substituting Eq. (18) into Eq. (16), and working out the trace, we obtain

$$\begin{aligned} P_{l_1 \rightarrow k_1}(x, k_{1T} < p_T) &= C_{q \rightarrow g} \frac{g^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \int_{\Lambda_{cutoff}^2}^{p_T^2} dk_{1T}^2 \frac{1}{k_{1T}^2} \\ &= \frac{N_c^2 - 1}{2N_c} \left(\frac{g^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \right) \ln \left(\frac{p_T^2}{\Lambda_{cutoff}^2} \right). \end{aligned} \quad (19)$$

Choosing $\bar{n} \cdot A = 0$ gauge, we derive the partonic scattering part of the $H(xl_1, l_2, p)$ defined in Eq. (10) from the diagram shown in Fig. 3,

$$\begin{aligned} \hat{H}(xl_1, l_2, p) &= g^4 \frac{1}{4} d_{\mu\nu} \text{Tr}(\gamma \cdot l_2 \gamma^{\beta'} \gamma \cdot (l_2 - k_2) \gamma^{\alpha'}) \\ &\quad \times P_{\rho\sigma}(p, \bar{n}) \frac{P_{\beta\beta'}(k_2, \bar{n})}{k_2^2} \frac{P_{\alpha\alpha'}(k_2, \bar{n})}{k_2^2} \\ &\quad \times \left[(-2xl_1 + p)^\sigma g^{\mu\beta} + (-2p + xl_1)^\mu g^{\beta\sigma} + (p + xl_1)^\beta g^{\sigma\mu} \right] \\ &\quad \times \left[(p - 2xl_1)^\rho g^{\alpha\nu} + (xl_1 + p)^\alpha g^{\nu\rho} + (-2p + xl_1)^\nu g^{\rho\alpha} \right], \end{aligned} \quad (20)$$

where $d_{\mu\nu}$ is defined as

$$d_{\mu\nu} = -g_{\mu\nu} + n_\mu \bar{n}_\nu + \bar{n}_\mu n_\nu. \quad (21)$$

In Eq. (20), the gluon polarization tensors are given by

$$\begin{aligned} P_{\rho\sigma}(p, \bar{n}) &= -g_{\rho\sigma} + \frac{p_\rho \bar{n}_\sigma + \bar{n}_\rho p_\sigma}{p \cdot \bar{n}}, \\ P_{\beta\beta'}(k_2, \bar{n}) &= -g_{\beta\beta'} + \frac{k_{2\beta} \bar{n}_{\beta'} + \bar{n}_\beta k_{2\beta'}}{k_2 \cdot \bar{n}}. \end{aligned} \quad (22)$$

Using the relations $k_2^2 = (p - xl_1)^2$ and $p^2 = 2p_+p_- - p_T^2 = 0$, we have

$$\frac{1}{k_2^2} = \frac{p_+}{xl_+} \frac{1}{p_T^2} \quad (23)$$

Substituting Eq. (23) into Eq. (20), and after some algebra, we obtain

$$\hat{H}(xl_1, l_2, p) = 4g^4 \left(\frac{p_+}{xl_+} \right)^2 \frac{1}{p_T^4} \left[(xs - 2xl_+p_-)^2 + (2xl_+p_-)(2p_+p_-) \right], \quad (24)$$

where $s = (l_1 + l_2)^2 = 2l_+l_-$ is the total invariant mass squared of the two incoming quarks. Substituting Eq. (24) into Eq. (20), and after taking into account of the color factor $1/2$, we obtain the hard scattering function $H(xl_1, l_2, p)$ as

$$\begin{aligned} H(xl_1, l_2, p) &= (2\pi)4g^4 \left(\frac{1}{2} \right) \left(\frac{p_+}{xl_+} \right)^2 \frac{1}{p_T^4} \frac{1}{s - 2l_+p_-} \delta(x - \frac{2p_+l_-}{s - 2l_+p_-}) \\ &\quad \times \left[(xs - 2xl_+p_-)^2 + (2xl_+p_-)(2p_+p_-) \right]. \end{aligned} \quad (25)$$

Combining Eq. (19), Eq. (25) and Eq. (8), we obtain

$$\begin{aligned}
E \frac{d\sigma_{qq \rightarrow g}}{d^3 p} &= \frac{g^6}{(2\pi)^4} \left(\frac{1}{2N_c} \right)^2 N_c (N_c^2 - 1) \\
&\times \int dx \delta(x - \frac{2p_+ l_-}{s - 2l_+ p_-}) (1 + (1 - x)^2) \left(\frac{p_+}{xl_+} \right)^2 \frac{1}{s(s - 2l_+ p_-)} \\
&\times \left[(s - 2l_+ p_-)^2 + \frac{2l_+ p_-}{x} (2p_+ p_-) \right] \left(\frac{1}{p_T^4} \right) \ln \left(\frac{p_T^2}{\Lambda_{cutoff}^2} \right). \quad (26)
\end{aligned}$$

We define the soft gluon limit as

$$\frac{p_-}{l_-} \ll 1 \quad \text{and} \quad \frac{p_+}{l_+} \ll 1. \quad (27)$$

At this soft gluon limit, we have $s = 2l_+ l_- \gg 2l_+ p_-$; and from the δ -function in Eq. (26), we have

$$x \approx \frac{p_+}{l_+} \ll 1, \quad \text{and} \quad 1 + (1 - x)^2 \approx 2. \quad (28)$$

Substituting Eq. (28) into Eq. 26, and taking the soft gluon limit, we obtain

$$E \frac{d\sigma_{qq \rightarrow g}}{d^3 p} = \frac{2g^6}{(2\pi)^4} \left(\frac{1}{2N_c} \right)^2 N_c (N_c^2 - 1) \left(\frac{1}{p_T^4} \right) \ln \left(\frac{p_T^2}{\Lambda_{cutoff}^2} \right). \quad (29)$$

Defining $y = \frac{1}{2} \ln(\frac{E+p_z}{E-p_z})$, we can rewrite the cross section in terms of variable y

$$\frac{d\sigma_{qq \rightarrow g}}{dy d^2 p_T} = \frac{2g^6}{(2\pi)^4} \left(\frac{1}{2N_c} \right)^2 N_c (N_c^2 - 1) \left(\frac{1}{p_T^4} \right) \ln \left(\frac{p_T^2}{\Lambda_{cutoff}^2} \right). \quad (30)$$

This is our final result. Eq. (30) shows the same p_T^2 dependence as the result obtained by Kovner, McLerran, and Weigert [5]. The difference between Eq. (3) is the factor of N_q^2 . Eq. (30) is obtained by calculating the leading contribution of the subprocess $qq \rightarrow qg$, for which N_q effectively equals to one. If we consider the total number of the quarks in the charge sources of both sides, we need to multiply N_q^2 to Eq. (30), and our result reproduces the result obtained by KMW.

IV. SUMMARY AND DISCUSSIONS

In this section we discuss the similarities and differences between KMW's result, Eq. (1) or Eq. (3), which was obtained in McLerran-Venugopalan formalism, and our result, Eq. (30), which was obtained in the conventional perturbative QCD formalism at the leading logarithmic approximation.

Our result can be reexpressed in terms of the usual factorized cross section in perturbative QCD. When we consider the collision between two nuclei, we can treat the two incoming quarks in Fig. 1 as coming from two nuclei respectively. In this picture, the number of the valence quark N_q is replaced by the quark distribution in the nuclei. The cross section in Eq. (8) (or equivalently Eq. (29)) is just the partonic cross section for the collision between

two quarks. In terms of the parton model, the cross section between the two nuclei A and B can be expressed in the following form:

$$E \frac{d\sigma_{AB \rightarrow g}}{d^3 p} = \int dz_1 dz_2 f_{q/A}(z_1) f_{q/B}(z_2) E \frac{d\sigma_{qg \rightarrow g}}{d^3 p}. \quad (31)$$

Here z_1 and z_2 are the momentum fractions of the quarks, and $f_{q/A}(z_1)$ and $f_{q/B}(z_2)$ are the quark distributions (or quark number densities) of the two nuclei. If we denote p_A and p_B as the momenta for the two nuclei respectively, then $z_1 = l_1/p_A$ and $z_2 = l_2/p_B$. Substituting Eq. (8) into Eq. (31), we have

$$E \frac{d\sigma_{AB \rightarrow g}}{d^3 p} \approx \frac{1}{2(2\pi)^3} \frac{1}{2S} \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} \left[\int \frac{dx_1}{x_1} f_{q/A}(z_1) f_{q/B}(z_2) P_{l_1 \rightarrow k_1}(x_1, k_{1T} < p_T) H(x_1 l_1, l_2, p) \right. \\ \left. + \int \frac{dx_2}{x_2} f_{q/A}(z_1) f_{q/B}(z_2) P_{l_2 \rightarrow k_2}(x_2, k_{2T} < p_T) H(l_1, x_2 l_2, p) \right] \quad (32)$$

where the overall factor 2 in Eq. (8) is now represented by the two terms, and $S = (p_A + p_B)^2 \approx 2p_A \cdot p_B$. In Eq. (32), $x_i = k_i/l_i$ with $i = 1, 2$, and $l_1 = z_1 p_A$ and $l_2 = z_2 p_B$. If we denote the momentum fraction of gluon k_1 with respect to p_A as $z'_1 = k_1/p_A$, and k_2 with respect to p_B as $z'_2 = k_2/p_B$, we can rewrite Eq. (32) in terms of z'_1 and z'_2 :

$$E \frac{d\sigma_{AB \rightarrow g}}{d^3 p} \approx \frac{1}{2(2\pi)^3} \frac{1}{2S} \int \frac{dz'_1}{z'_1} \frac{dz'_2}{z'_2} \\ \times \left\{ \left[\int \frac{dz_1}{z_1} f_{q/A}(z_1) P_{l_1 \rightarrow k_1}(z'_1/z_1, k_{1T} < p_T) \right] \right. \\ \times \left(\int \frac{dz_2}{z_2} f_{q/B}(z_2) \delta \left(1 - \frac{z'_2}{z_2} \right) \right) \\ + \left(\int \frac{dz_1}{z_1} f_{q/A}(z_1) \delta \left(1 - \frac{z'_1}{z_1} \right) \right) \\ \left. \times \left[\int \frac{dz_2}{z_2} f_{q/B}(z_2) P_{l_2 \rightarrow k_2}(z'_2/z_2, k_{2T} < p_T) \right] \right\} H(z'_1 p_A, z'_2 p_B, p) \quad (33)$$

$$\approx 2 \left(\frac{1}{2(2\pi)^3} \frac{1}{2S} \right) \int \frac{dz'}{z'} \frac{dz}{z} \left[\int \frac{dz_1}{z_1} f_{q/A}(z_1) P_{l_1 \rightarrow k_1}(z'/z_1, k_{1T} < p_T) \right] \\ \times f_{q/B}(z) H(z' p_A, z p_B, p). \quad (34)$$

In obtaining Eq. (34), we used the fact that the partonic scattering part $H(k_1, k_2, p)$ in Eq. (33) is symmetric under the exchange of the k_1 and k_2 at the soft gluon limit.

According to the QCD factorization theorem [1], we see that the part inside the square brackets is actually the gluon distribution from nuclei A (or B) at the factorization scale $\mu_F^2 = p_T^2$, with only the quark splitting function [10],

$$f_{g/A}(z'_1, \mu_F^2 = p_T^2) = \int \frac{dz_1}{z_1} f_{q/A}(z_1) P_{l_1 \rightarrow k_1}(z'_1/z_1, k_{1T} < p_T) \\ + \text{term from gluon splitting.} \quad (35)$$

Using Eq. (35), we can then reexpress Eq. (33) as:

$$E \frac{d\sigma_{AB \rightarrow g}}{d^3 p} \approx \frac{1}{2(2\pi)^3} \frac{1}{2S} \int \frac{dz'}{z'} \frac{dz}{z} \left[f_{g/A}(z', \mu_F^2 = p_T^2) f_{q/B}(z) H(z' p_A, z p_B, p) + f_{q/A}(z) f_{g/B}(z', \mu_F^2 = p_T^2) H(z p_A, z' p_B, p) \right], \quad (36)$$

which is the factorized formula for two-to-two subprocesses in the conventional perturbative QCD for the nucleus-nucleus collisions. In KMW formalism, only the valence quark color charge was used as the source of the classical charge of colors. As a result, the gluon splitting term in Eq. (35) is neglected for the distribution $f_{g/A}$.

Our discussions above show that the soft gluon distribution in heavy ion collisions obtained in KMW's approach can be understood by calculating the partonic process $qq \rightarrow qgg$. To relate KMW's result to the factorized formula in the conventional perturbative QCD, we need to: (1) replace the charge density for the classical color charge μ^2 (or equivalently N_q) by the valence quark distributions of the nuclei; (2) absorb the logarithm $\ln(p_T^2/\Lambda_{cutoff}^2)$ into one of the valence quark distributions, which effectively becomes the gluon distribution of one of the initial nuclei. From Eq. (33), we identify that the $\ln(p_T^2/\Lambda_{cutoff}^2)$ factor in Eq. (1) (or in Eq. (3)) comes from the logarithm of the splitting of the incoming quark to the soft gluon in Fig. 2. In terms of the conventional QCD factorization theorem [1], such logarithm is normally factorized into the distributions of the collinear gluons inside the incoming hadrons, as demonstrated in Eq. (35), and the logarithmic dependence of the distributions is a direct result of solving the DGLAP evolution equations [8].

From the above comparison, we conclude that by solving the classical Yang-Mills Equation to the second order in iteration, KMW's result reproduces the result of conventional perturbative QCD at the leading logarithmic approximation, with the convolution over the parton number densities inside the nuclei replaced by the effective numbers of the valence quarks. The logarithmic dependence shown in KMW's result basically describes the logarithmic DGLAP evolution of the quark distributions. However, in addition to the valence quarks, the glue field at small x can be produced by *all* flavor partons that have larger momenta.

The McLerran-Venugopalan formalism was later further developed by Ayala, Jalilian-Marian, Kovner, McLerran, Leonidov, Venugopalan, and Weigert [11,12]. The major improvement to the McLerran-Venugopalan model is to include the harder gluons into the charge density μ^2 and treat the charge source as an extended distribution which depends on the rapidity [11]. These improvements lead to the “renormalization” of the charge density. It was showed that the renormalization group equation for the charge density can be reduced to the BFKL equations [13] in some appropriate limits [12]. It will be very interesting to see if KMW's approach, after including higher orders of iteration, can show the parton recombination [14] and other non-perturbative effects which are not apparent in the normal perturbative calculation.

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FIGURES

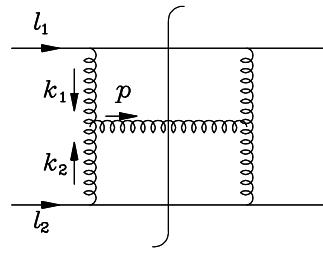


Fig. 1

FIG. 1. Square of the leading Feynman diagram to the process: $qq \rightarrow qqq$.

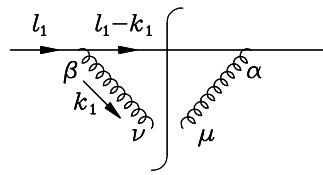


Fig. 2

FIG. 2. Feynman diagram for the splitting of $q \rightarrow g$.

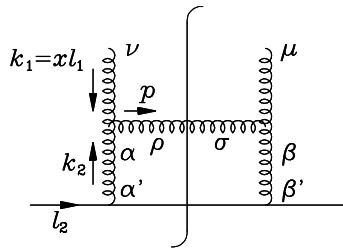


Fig. 3

FIG. 3. Leading Feynman diagram contributing to the hard scattering part $H(xl_1, l_2, p)$.

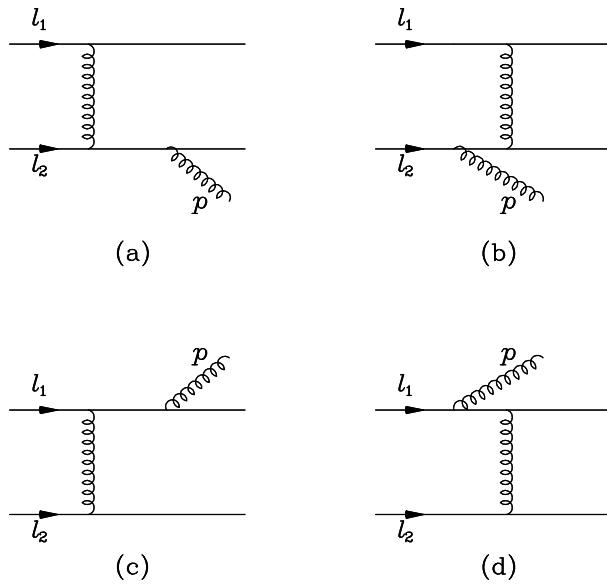


Fig. 4

FIG. 4. The rest Feynman diagrams to the process: $qq \rightarrow qgg$, in addition to the diagram in Fig. 1.

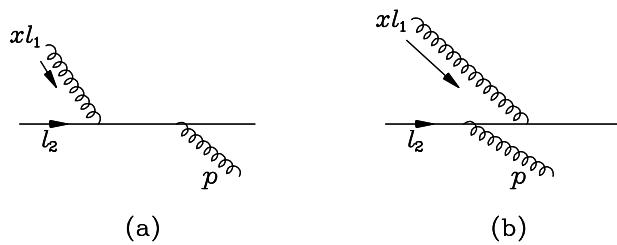


Fig. 5

FIG. 5. Feynman diagrams contributing to $H_a(xl_1, l_2, p)$ (a), and $H_b(xl_1, l_2, p)$ (b).